

# CAS Screenshot Questions

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I teach at a school in which all students from Year 7 onwards have a Computer Algebra System (CAS) calculator; we prescribe it on the school booklist. CAS is perhaps the best and most important of all technologies for the mathematics classroom. At the speed of light, it can process much of what we teach to our students. Despite its awesome power, CAS does not diminish or replace existing content but it does open up new possibilities for mathematics learning and for the assessment of learning.

To understand CAS, students must first learn mathematics. To use it, they must learn how to “drive” their calculator and become fully conversant with CAS syntax. Importantly, though, they must not use CAS syntax in their handwritten or “pen and paper” mathematics.

The use of a CAS calculator in multiple-choice questions and in extended response test items is already well-developed. In Victoria, for example, some of the examinations in two of the three Year 12 Mathematics subjects are CAS-dependent papers. In this article, though, my focus is on the use of CAS in non-CAS test papers, in what I call CAS screenshot questions. By this, I mean the inclusion of test items in which candidates are provided with a CAS screenshot and in which they are required to employ “pen and paper” methods to obtain the correct output that would otherwise have been provided by the machine.

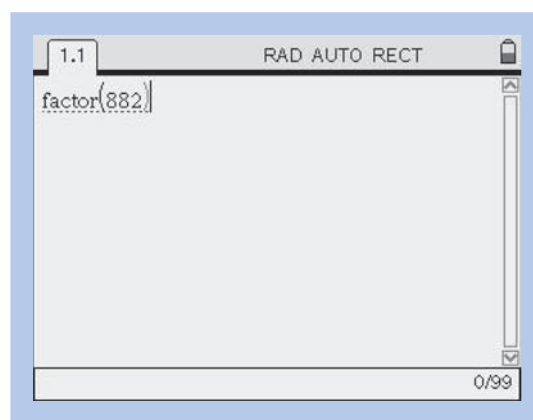
Many CAS dependent test items require students to do little more than merely “push buttons” to produce the correct answer. The advantage of CAS screenshot questions in non-CAS test papers is that the students must recognise the mathematical context of the question and be fluent in the use of any algorithms required to solve it, in addition to having a thorough knowledge of CAS functions and syntax.

The following examples are typical of the CAS screenshot items I’ve written for non-CAS test papers. Some of my examples may just as readily be incorporated into test papers using non-CAS technologies.

To understand CAS, students must first learn mathematics.

## Example 1: Year 7 Mathematics

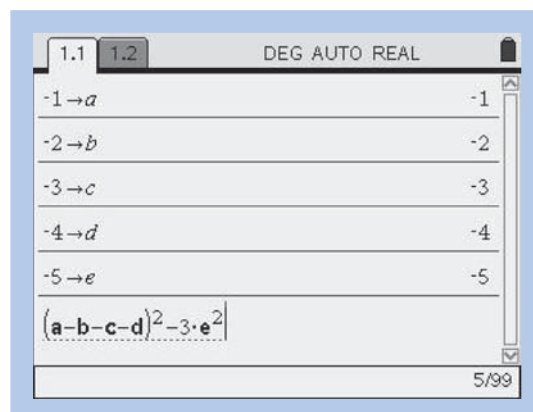
Examine the CAS screen shown below. Find the output, given by the machine, after pushing the enter key.



In this question, students must first know that “factor” in CAS takes the number supplied inside the brackets and rewrites it as a product of its prime factors in index form. To successfully answer the question, students need to use a factor tree, or an equivalent method, to find the prime factors using “pen and paper” mathematics. Those who successfully factorise the number 882 but fail to write their answer in index form, will be awarded the question’s method mark but they will not receive the allocated answer mark.

## Example 2: Year 8 Mathematics

Examine all six lines on the CAS screen shown below. Note the position of the cursor at the end of the sixth line. Find the output, given by the machine, after pushing the enter key.



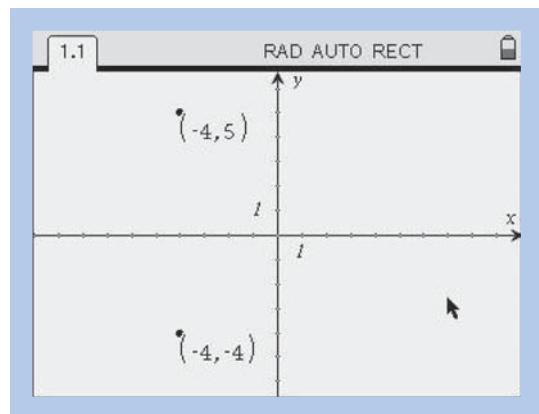
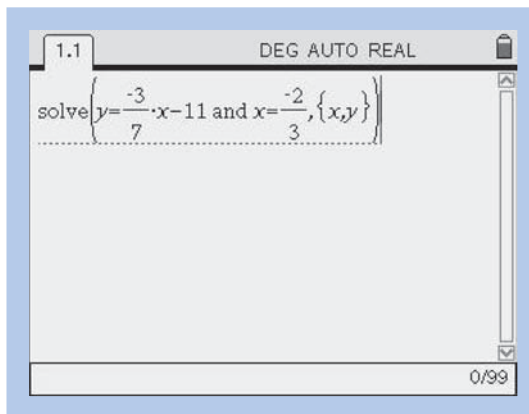
We rarely test students across multiple topics. The tyranny of the topic test too often dominates our thinking!

In this question, students must understand the store facility of CAS and how the machine will substitute the stored values into the sixth line. They also need to realise that CAS employs a screenshot “dot” for multiplication. The “pen and paper” mathematics involved here is quite complex for Year 8 students, requiring skills in both integer arithmetic and order of operations.

It is worth noting here that students should always include multiplication signs and brackets when using CAS.

**Example 3: Year 8 Mathematics**

Examine the CAS screen shown below.



- a. Write down the coordinates of a possible third ordered pair to complete a right-angled isosceles triangle.
- b. Hence find the area of the right-angled isosceles triangle so formed.

This example tests higher order thinking skills, as well as an ability to synthesise knowledge across geometry, coordinate geometry and measurement. As teachers we rarely test students across multiple topics. The tyranny of the topic test too often dominates our thinking! In a later article I will introduce readers to the concept of cumulative testing.

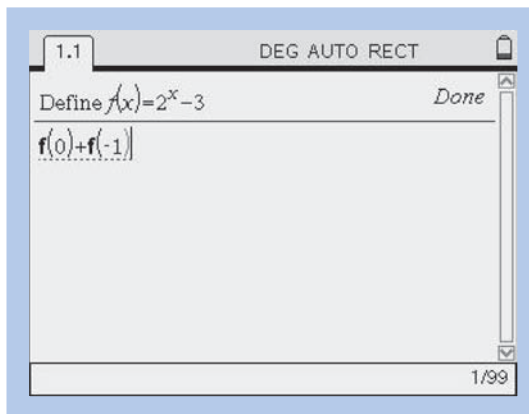
**Example 4: Year 9 Mathematics**

Examine the CAS screen shown below. Find the output, given by the machine, after pushing the enter key.

Students who find the correct  $y$  coordinate of the point of intersection but who neglect to also state the  $x$  coordinate of the same point in their answer will not receive the question’s allocated answer mark.

**Example 5: Year 10 Mathematics**

Examine the CAS screen shown below. Find the output, given by the machine, after pushing the enter key.

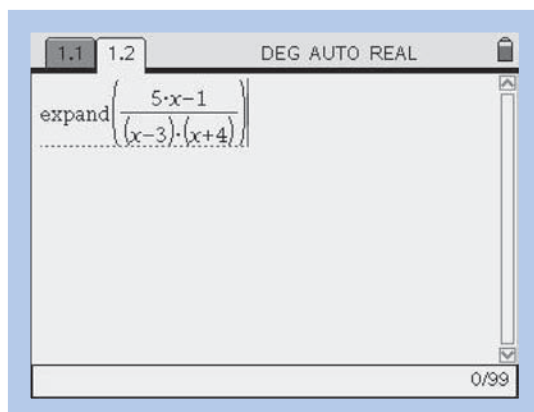


The CAS facility to define functions and to be able to recall them at will, across multiple pages and documents, is a very powerful investigative tool!

**Example 6: Year 10 Mathematics**

Examine the CAS screen shown below. Find the output, given by the machine, after pushing the enter key.

The CAS facility to define functions and to be able to recall them at will, across multiple pages and documents, is a very powerful investigative tool!



Students are generally very good at remembering that “expand means remove brackets”, and that “expand is the opposite of factorise”. In this example, they need to think more broadly about the instruction to expand.

We spend a lot of time getting our students to practise adding and subtracting fractions using a lowest common denominator. Rarely do we invite them to go the other way and split a single fraction into its constituent parts. Why would we ever want to “complicate” a fraction in such a way? Well, in later years, the ability to write a fraction as a sum of its partial fractions enables us to then integrate the original fraction.

Partial fractions, hard as they are to teach to Year 10 students, introduce the concept of an identity, as opposed to a mere equation. Such an introduction paves the way for the later appearance of the Pythagorean Identity in circular functions.

The structure and composition of CAS screenshot test items is limited only by our imagination and skill as teachers. Their use opens up a plethora of new possibilities for the assessment of student learning in mathematics. And many of the possibilities examine higher order thinking skills.

All of the screenshots in my examples were produced using software that links my handheld CAS calculator to my computer. Please note that I allocate only a small proportion of the available marks on each non-CAS test to CAS screenshot items. This proportion is usually around ten per cent, which is sufficient to provide students with

an incentive to regularly practise and master their CAS skills, over and above their other more traditional skills in the subject. The ten per cent of marks estimate approximates the proportion of actual class time I spend teaching and developing CAS skills.

### Contentment

Space and number,  
Reasoning and data.  
Change and approximation,  
Pencil and paper.

Functions and relations,  
One-sided operator.  
Exact solutions,  
Symbolic calculator.

Conjugate hyperbolae,  
Perpendicular lines.  
Double angle formulae,  
Sines or cosines?

Cyclic permutation,  
Rational denominator.  
Complementation,  
Population estimator.

Curve of best fit,  
Difference of squares.  
Times and tip,  
Disjoint pairs.

Cartesian plane,  
Recurring decimal.  
Projectile range,  
Infinitesimal.

Efficient algorithm,  
Constant acceleration.  
Natural logarithm,  
Long multiplication.

Null factor law,  
Perfect cubes.  
Vectorial tensor,  
Order of magnitude.

Segment or sector?  
Distance or displacement?  
Scalar or vector?  
Lifetime contentment!

The structure and composition of CAS screenshot test items is limited only by our imagination and skill as teachers.

### About the author

Russell Boyle is an author, teacher and poet. He has a Baccalaureate in pure mathematics and a Master's degree in education. He commenced his teaching career at Braybrook High School in 1975. Four years later he transferred to Lilydale High School. Between 1983 and 1985 he taught at Mentone Boys' Grammar School, and moved to St. Paul's Anglican Grammar School in 1986, as head of mathematics. Russell returned to the public sector in 2001, as head of mathematics at Glen Waverley Secondary College. Since 2003 he has been dean of mathematics at Ruyton Girls' School. Russell has published three volumes of poetry, authored five web programming courses and two advanced course exams. His school performance tables were published by the *Herald Sun* between 2002 and 2006. Russell may be contacted through his website at russellboyle.com