

Cumulative Testing

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Introduction

As teachers, we rarely test across multiple topics. The tyranny of the topic test will often dominate our thinking. In this article I discuss my use of “cumulative testing” and provide an annotated example of a cumulative test, written for and administered to a Year 8 Mathematics class.

According to the Oxford English Dictionary, the word “cumulative” means increasing in quantity or degree by successive additions. Mathematics is the most sequential of all the academic disciplines. “Cumulative” is therefore the perfect adjective for “testing” in our subject.

My first attempts at writing cumulative test papers were not particularly adventurous. They involved the addition of revision questions at the end of each topic test. I gave no information as to which of our earlier topics might be included; so students had to prepare thoroughly across all topics studied. This encouraged them to develop and sustain a home revision program, given that to secure a high test grade they needed to solve the revision questions, in addition to those questions which addressed the current topic.

With practice my writing and test design skills have improved and so too has the level of sophistication of my cumulative tests. I aim to test multiple concepts, to integrate past work with new work. I strive to produce papers that are reliable and valid as measurement instruments. My greatest criticism of many mathematics test papers, especially commercially prepared tests, is that while most are reliable instruments, in that they measure consistently whatever it is they test, few actually measure what they purport to test.

I spend three to four hours formulating, writing and formatting each cumulative test. Unfortunately, the time and effort devoted to this important area of my teaching practice has, throughout my career, gone unacknowledged and unrewarded by my senior managers and principals. It is therefore not surprising to find that, rather than write their own test papers, many teachers turn instead to commercially prepared and often poorly written test items. Worse still, some choose to administer a reprint of last year’s test, without reflection or amendment.

A suggested rationale for cumulative testing is to:

- Encourage students to view their learning holistically, rather than to merely focus on the current unit of work;
- Persuade students of the value and importance of regular revision;
- Enable students to better see how discrete concepts and algorithms mesh, over time, into a sequential whole;
- Increase the academic rigour of school mathematics;
- Better prepare students for examinations, which in our subject cover the entire course and assume knowledge of material from past years.

What follows is an annotated example of a cumulative test, written for my Year 8 Mathematics class. The topic was Equations which followed our earlier work on Integers and real numbers, Set notation and theory, Percentages, and Algebra.

Year 8 equations cumulative test (no CAS, no reference materials)

1. Solve each of the following equations for x

a. $x + 3.07 = 1.2$

$$x = 1.2 - 3.07$$

$$x = -1.87$$

b. $0.03x = -4.8$

$$x = -\frac{4.8}{0.03}$$

$$x = -\frac{480}{3}$$

$$x = -160$$

c. $\frac{x}{-5} = -1\frac{2}{15}$

$$x = -\frac{17}{15} \times -\frac{5}{1}$$

$$x = \frac{17}{3}$$

$$x = 5\frac{2}{3}$$

The old school textbooks Victorian schools *Arithmetic for Grades III to VIII* contain Cumulative Revision tests.

Careful drafting of test questions that are similar but not repetitive can focus diagnostically on specific concepts and processes.

d. $x - 1\frac{5}{6} = -2\frac{1}{7}$
 $x = -\frac{15}{7} + \frac{11}{6}$
 $x = \frac{-90 + 77}{42}$
 $x = -\frac{13}{42}$

e. $-2x - 17 = -58$
 $-2x = -58 + 17$
 $-2x = -41$
 $x = \frac{-41}{2}$
 $x = 20\frac{1}{2}$

f. $13(4 - x) = 91$
 $4 - x = 7$
 $-x = 3$
 $x = -3$

g. $\frac{3x - 5}{7} = -3.5$
 $3x - 5 = -24.5$
 $3x = -24.5 + 5$
 $3x = -19.5$
 $x = -6.5$

h. $\frac{4x}{7} - 5 = -3.5$
 $\frac{4x}{7} = -3.5 + 5$
 $4x = 1.5 \times 7$
 $x = \frac{10.5}{4}$
 $x = 2.625$

i. $3x - 7 - 9x + 11 = -5$
 $-6x + 4 = -5$
 $-6x = -9$
 $x = \frac{-9}{-6}$
 $x = \frac{3}{2}$
 $x = 1\frac{1}{2}$

j. $\frac{23x}{6} = 142.6$
 $23x = 855.6$
 $x = 37.2$

(10 × 2 = 20 marks)

Author's notes:

- In all parts of question **1**, students are required to demonstrate the ability to add, subtract, multiply and divide positive and negative fractions and decimals, as well as the skill to use inverse operations to solve linear equations.
- I deliberately included similar operations and numbers in the equations of questions **1g** and **1h**, so I could discriminate between those students who knew the correct order of inverse operations from those who didn't.
- In question **1i** students must first be able to collect like terms.
- The long division I wanted students to use in question **1j** may be regarded by some teachers as an old-fashioned algorithm but to me it is a necessary skill for later algebraic work with the Factor Theorem and for the sketching of rational functions in coordinate geometry.

2. Solve each of the following equations for x

a. $3x - 3 = 5x + 11$
 $-3 = 2x + 11$
 $-14 = 2x$
 $x = -7$

b. $2(x - 7) - 9(x - 6) = 3(x + 2) - 56$
 $2x - 14 - 9x + 54 = 3x + 6 - 56$
 $-7x + 40 = 3x - 50$
 $40 = 10x - 50$
 $90 = 10x$
 $x = 9$

(2 + 3 = 5 marks)

Author's note:

- In question **2b** students must first be able to expand brackets, and make the sign change when the coefficient of the bracket is negative.

Allocating marks to a test question can be an arbitrary convention, or can allow 1 mark for each expected computational step.

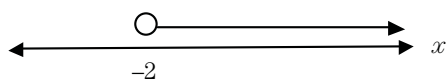
Correctly using filled or open end-points in number-line diagrams helps students grasp the difference between non-including “less than” contrasted with inclusive “less than or equal to”.

3. Find $\{x : 7x + 11 > -3, x \in R\}$ and graph the solution set on a number line.

$$7x > -14$$

$$x > -2$$

$$\text{Solution set} = \{x : x > -2, x \in R\}$$



(3 marks)

4. Find $\{x : \frac{71-x}{7} \geq 9, x \in R\}$ and graph the solution set on a number line.

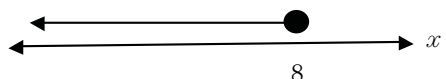
$$71 - x \geq 63$$

$$-x \geq 63 - 71$$

$$-x \geq -8$$

$$x \leq 8$$

$$\text{Solution set} = \{x : x \leq 8, x \in R\}$$



(3 marks)

Author’s note:

- In questions 3 and 4 an understanding of set notation is essential, as is the ability to solve linear inequations, including the special case when the coefficient is negative. Real number solution sets mean the number line graphs must be continuous with an open or closed circle at one end and an arrow at the other.

5. Four consecutive even integers sum to -260 . Find the largest of the four integers.

$$\text{Let } x \text{ be the smallest integer}$$

$$x + (x + 2) + (x + 4) + (x + 6) = -260$$

$$4x + 12 = -260$$

$$4x = -272$$

$$x = -68$$

$$\text{The largest consecutive integer is } -62$$

(3 marks)

Author’s note:

- In question 5 some students defined x as the largest integer; most went on to produce a correct solution which was more efficient than mine.

6. A Holden Commodore has lost 86% of its original value over the past ten years. If the market valuation of the Holden Commodore is now \$4900, how much did it cost to purchase as a new car ten years ago?

$$\text{Let } x \text{ be the new car purchase cost}$$

$$14\% \text{ of } x = \$4900$$

$$0.14x = 4900$$

$$x = \frac{4900}{0.14}$$

$$x = \frac{490000}{14}$$

$$x = 35000$$

The cost of the car ten years ago was \$35 000

(3 marks)

Author’s notes:

- A sound knowledge of percentages together with an ability to apply that knowledge to a non-routine problem is required in question 6.
- Rather than use an equation, some students preferred to use the unitary method in this question.

7. The length of a rectangle is x cm. The width of the rectangle is 9 cm shorter than its length.

- a. Write down an expression, in terms of x , for the width of the rectangle.

$$x - 9$$

- b. If the perimeter of the rectangle is 108 cm, then write down an equation and solve for x .

$$2x + 2(x - 9) = 108$$

$$2x + 2x - 18 = 108$$

$$4x - 18 = 108$$

$$4x = 126$$

$$x = 31\frac{1}{2} \text{ cm}$$

(1 + 2 = 3 marks)

Author’s note:

- Extended-response question 7 assumes students are familiar with the concept of perimeter from Year 7 Mathematics.

Understanding percentages as fractional parts of a quantity links with earlier ideas of multiplication as “so many repeated lots of a given quantity”.

It is crucial that HOW students are TESTED is based on how they are TAUGHT.

8. The formula $S = 180(n - 2)$ can be used to calculate the sum of the interior angles, S , of a regular polygon with n sides.

a. Find the sum of the interior angles in a polygon with 25 sides.

$$S = 180(25 - 2)$$

$$S = 180 \times 23$$

$$S = 18 \times 10 \times 23$$

$$S = 414 \times 10$$

$$S = 4140^\circ$$

b. Find the number of sides in a polygon whose interior angles sum to 6660°.

$$6660 = 180(n - 2)$$

$$\frac{6660}{180} = n - 2$$

$$37 = n - 2$$

$$39 = n$$

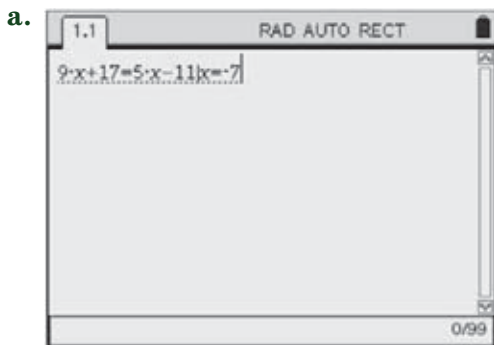
The polygon has 39 sides

(2 + 2 = 4 marks)

Author's note:

- The formula was provided in the stem of question 8 because at the time of the test we hadn't studied geometry. When we did, later in the year, this question served as a good introduction to the angle sum of a polygon.

9. Examine the CAS screens shown below. In each case, find the output, given by the machine, after pushing the enter key.

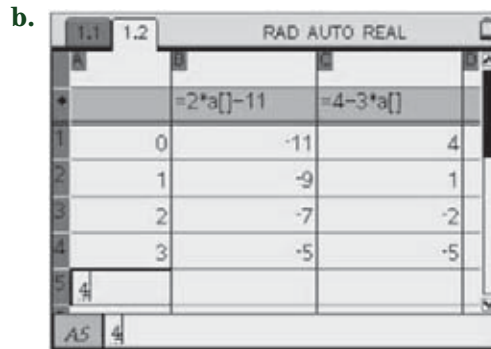


$$4x + 17 = -11$$

$$4x = -28$$

$$x = -7$$

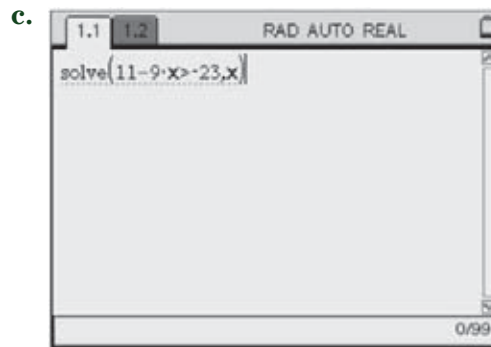
Output will be 'true'



$$2 \times 4 - 11 = 8 - 11 = -3$$

$$4 - 3 \times 4 = 4 - 12 = -8$$

Output in row 5 will be: 4 -3 -8



$$-9x > -23 - 11$$

$$-9x > -34$$

$$x < \frac{-34}{-9}$$

Output will be $x < \frac{34}{9}$

(2 + 2 + 2 = 6 marks)

Author's note:

- I limit CAS screenshot questions to around 10 per cent of the available marks, which approximates the proportion of class time I devote to the teaching of CAS skills.

End of test

The only certainty

The only hope,
is love.
The only faith,
is trust.
The only grace,
is humility.

Do you expect students to MEMORISE definitions or formulas that have been studied? If so, they do not need to be explained or provided in a test question. But if not, then provide any necessary background knowledge as a reminder.

Working in classrooms with students is like an on-going conversation about the topic at hand — mathematics and its applications. Sometimes this leads me to “reflect” on these conversations, in poetic ways.

The greatest strength,
is compassion.

The greatest need,
is tolerance.

The greatest polity,
is freedom.

The only information,
is truth.

The only knowledge,
is wisdom.

And the only certainty,
is uncertainty.

About the author

Russell Boyle is an author, teacher and poet. He has a baccalaureate in pure mathematics and a master’s degree in education. He is dean of mathematics at Ruyton Girls’ School; his previous positions include head of mathematics at Glen Waverley Secondary College and head of

mathematics at St Paul’s Anglican Grammar School. Russell has authored five web programming courses and two advanced course exams. He has published three volumes of poetry. “The Only Certainty” is from his first anthology. Russell may be contacted through his website at russellboyle.com

In the eyes of their students, teachers may seem to know everything. But the more we teachers do learn, the more we realise we do not yet know ...